A Random Random Walk Walk

Derek Lim
MIT Splash! 2022

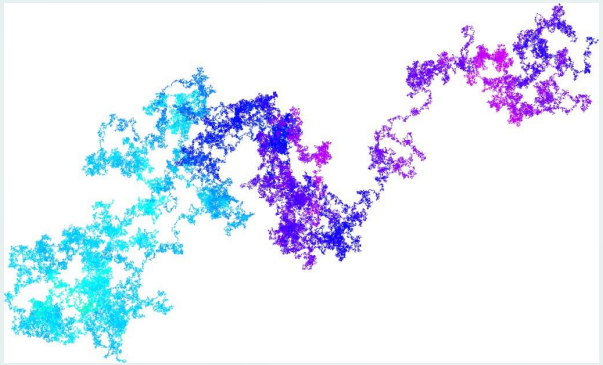
Outline

- What is a random walk?
- Random walks in 1d vs 2d vs 3d (Math)
- Continuous random walks + Brownian Motion (Math, physics)
- PageRank algorithm (Computer science)
- Levy flight and animal motion (Biology, ecology)

What is a Random Walk?

What is a random walk?

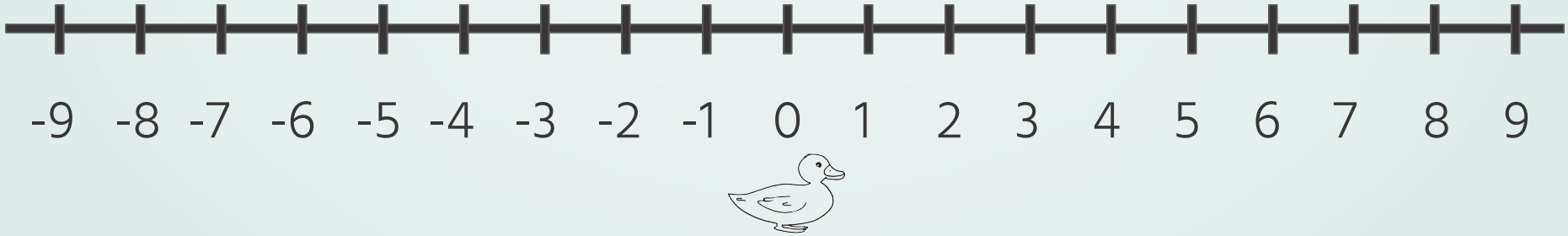
Definition: Something that selects a random place to move to at each time step



(Definition can be made more precise with probability theory)

Example 1: simple 1D random walk

A duck moves left or right with equal probability



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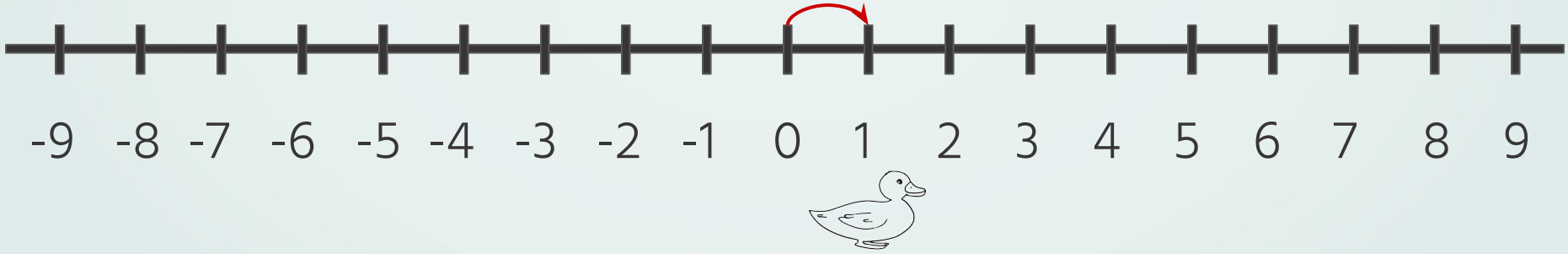

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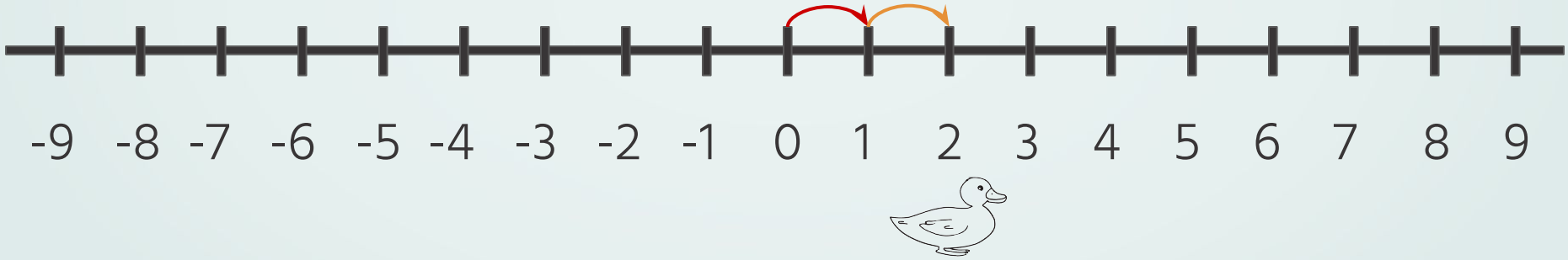

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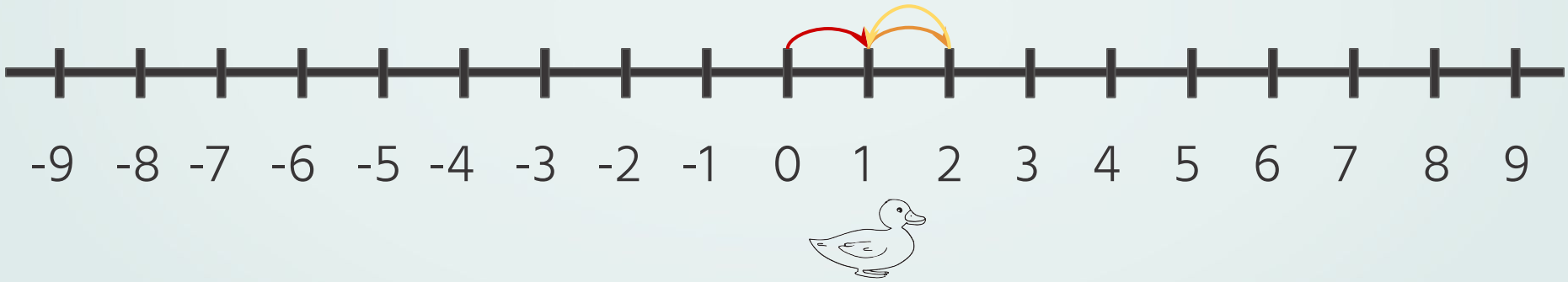

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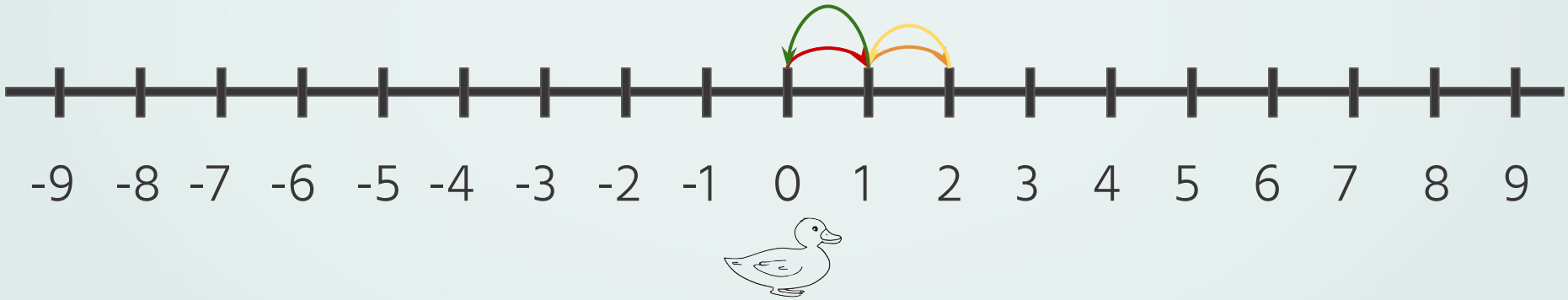

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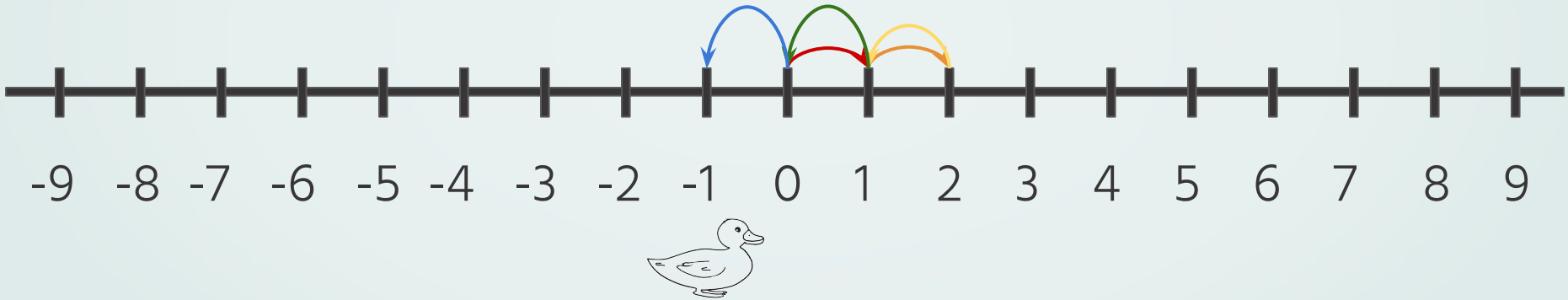

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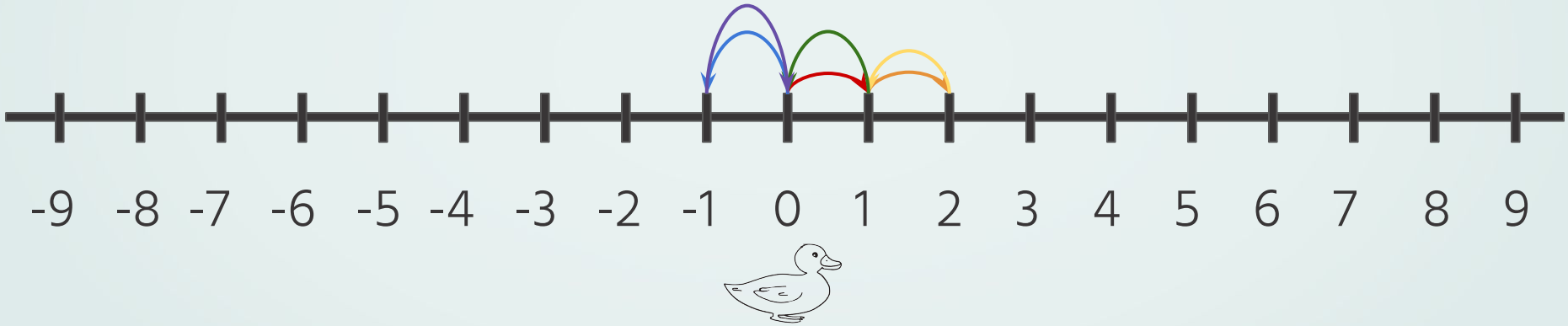


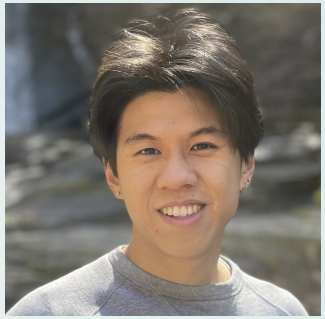
Example 2: Random walk in Manhattan

Why do we care about random walks?

- They model many real-world processes (will see today)
- Very interesting mathematical objects
- Useful for algorithms and computer science (related to my research!)

About me

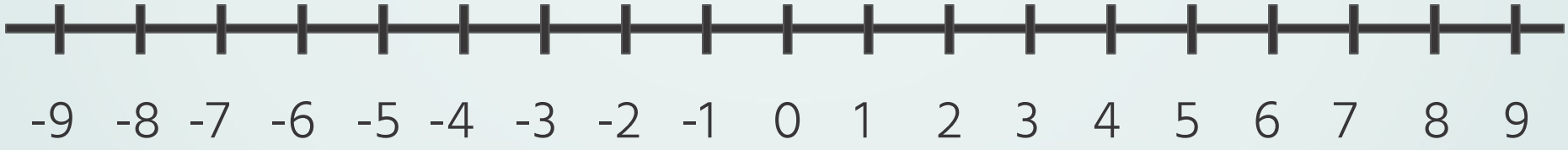
- Second year PhD student in computer science at MIT, doing AI research
- Studied math and computer science during undergrad at Cornell
- Mix of theory and application in my research



1D, 2D, and 3D simple random walks

Recall the simple 1D random walk

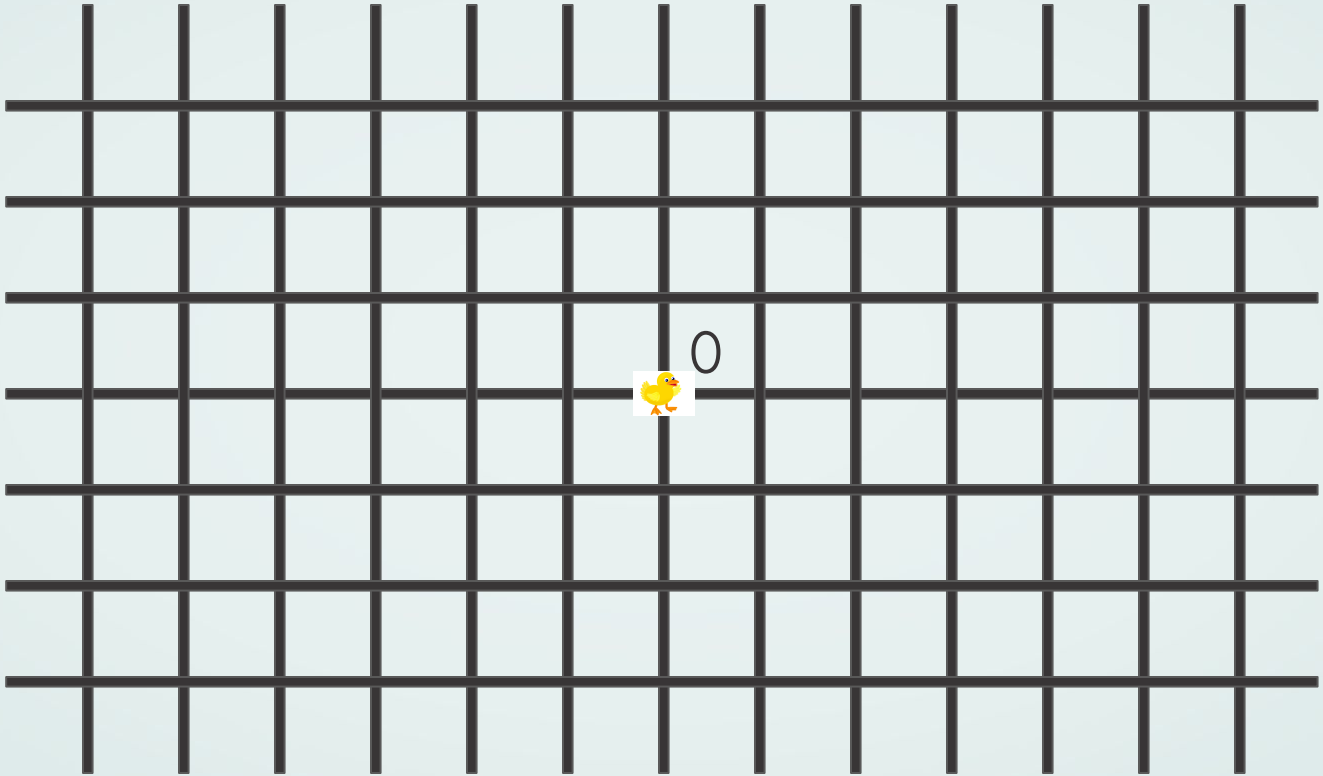
Flip a coin, move right if heads, move left if tails



Simple 2D random walk

Flip 2 coins in sequence

HH=up
HT=right
TT=down
TH=left



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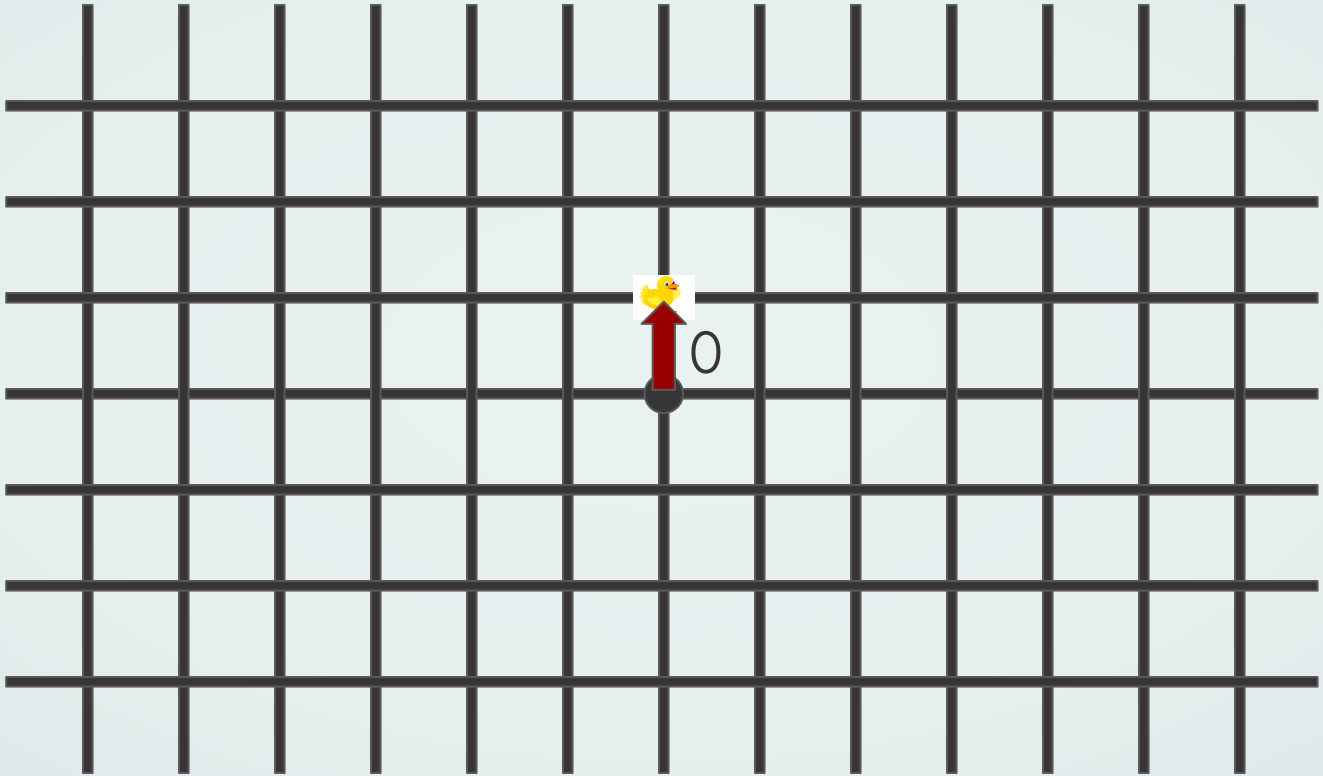

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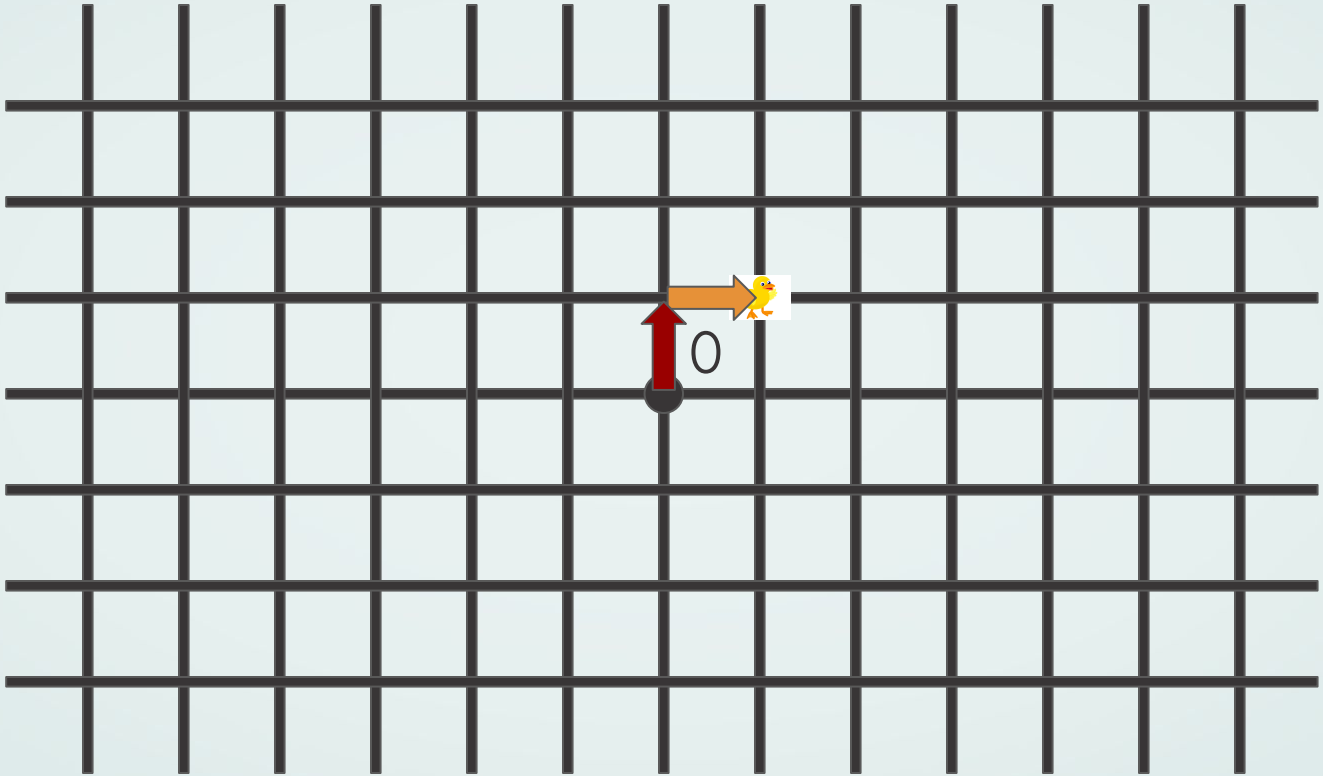

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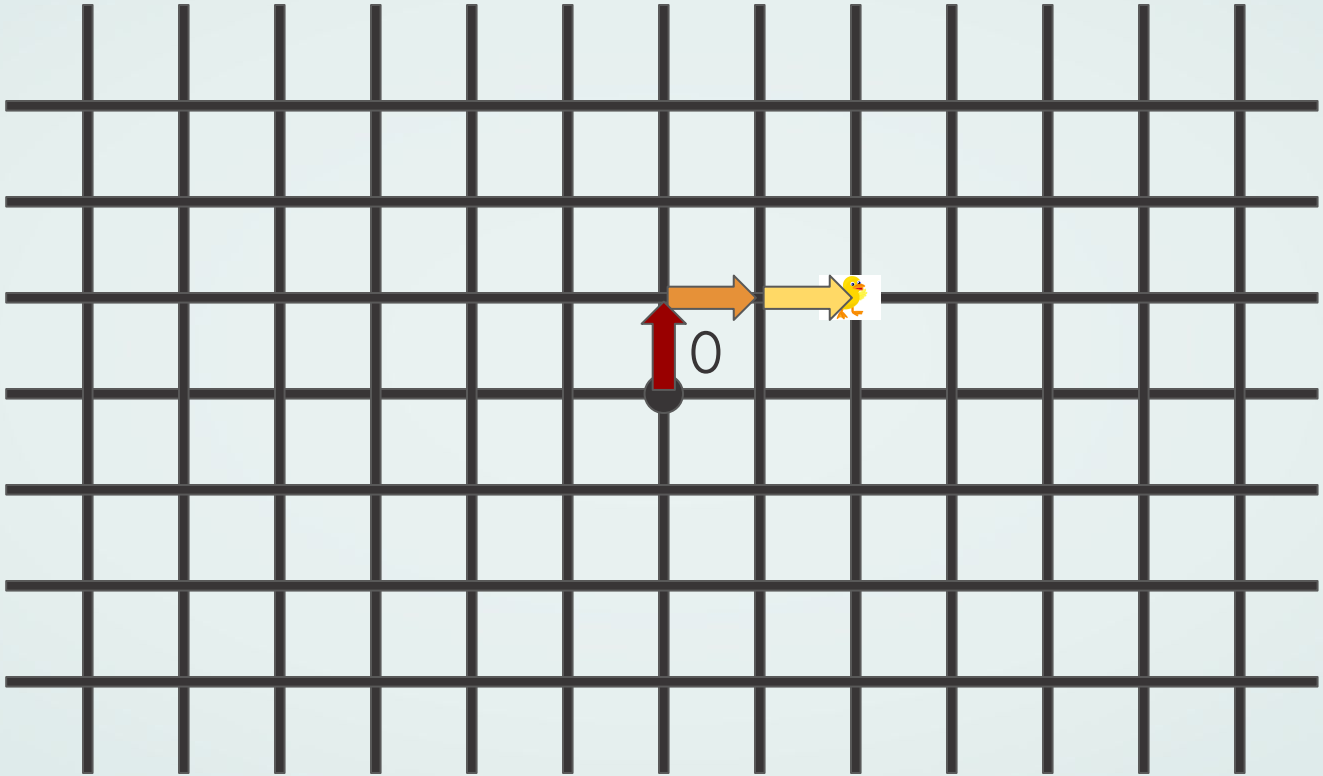

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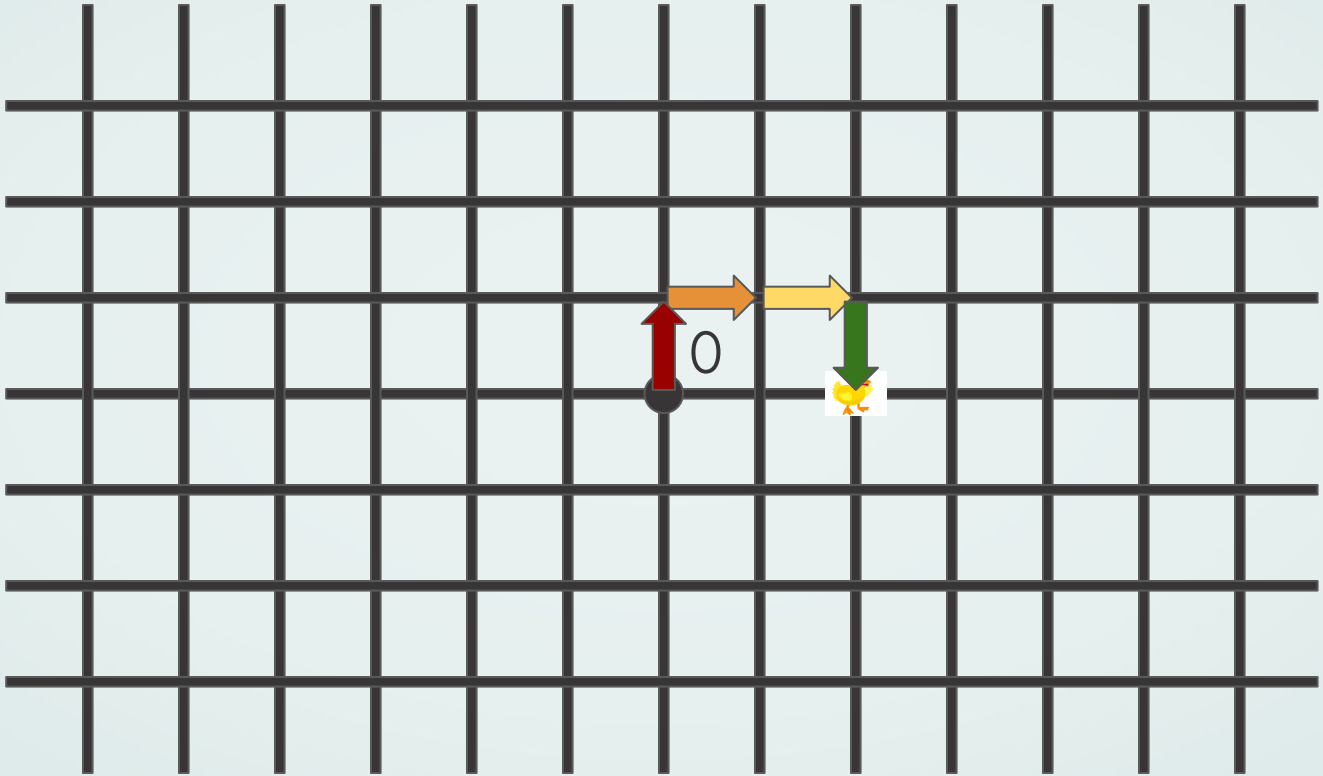

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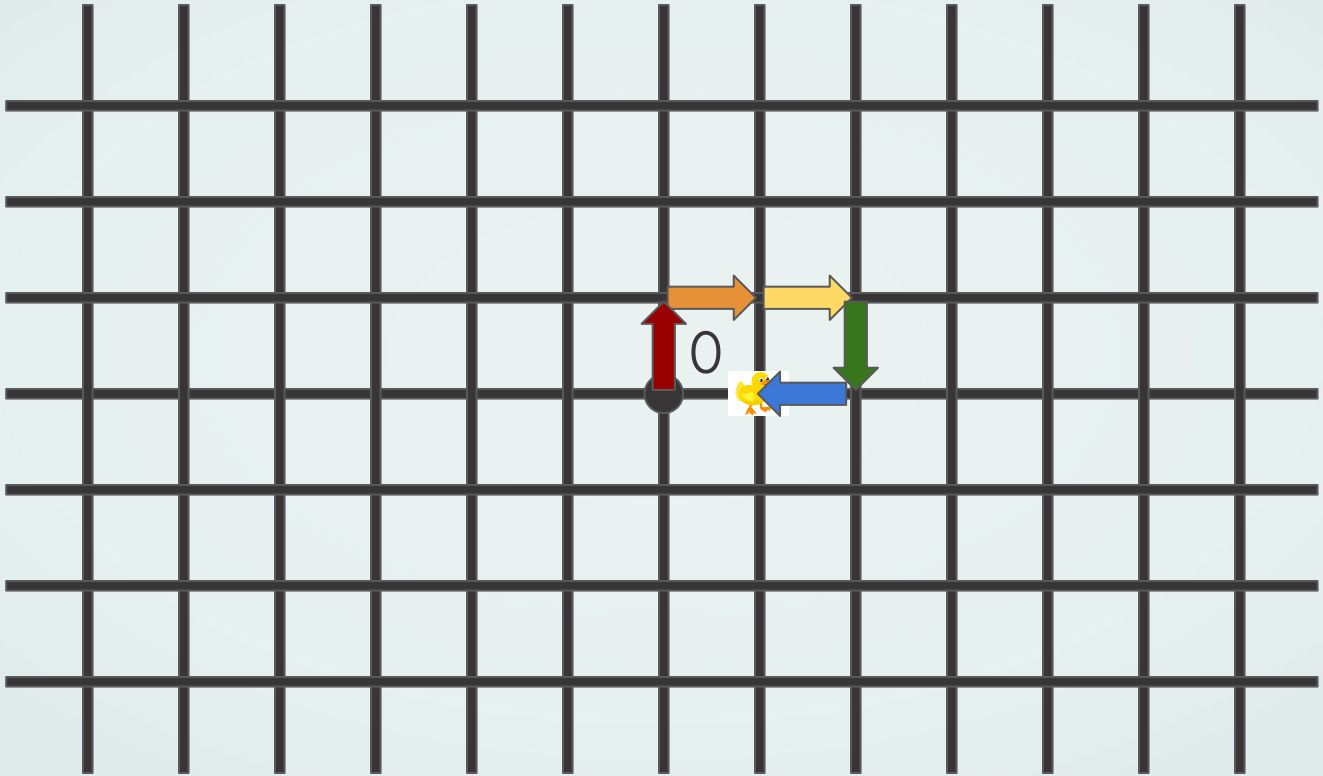

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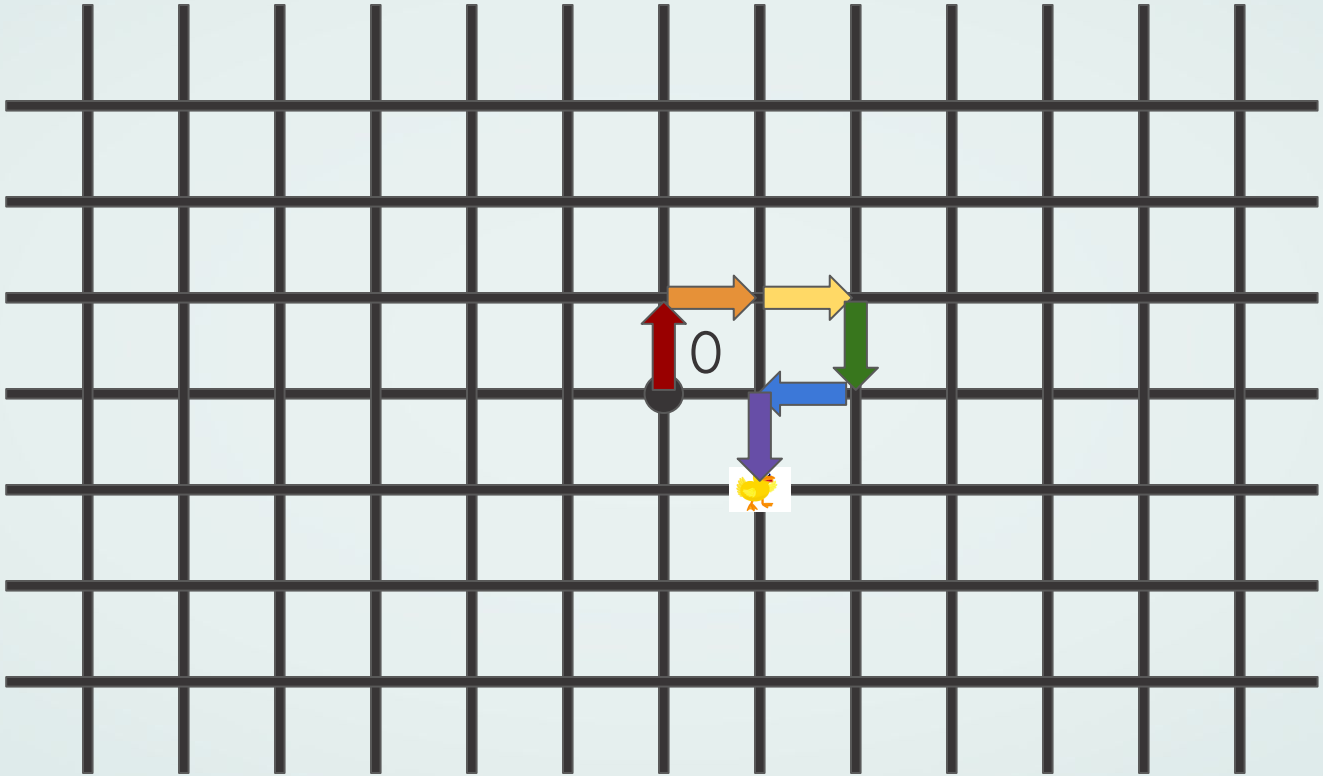

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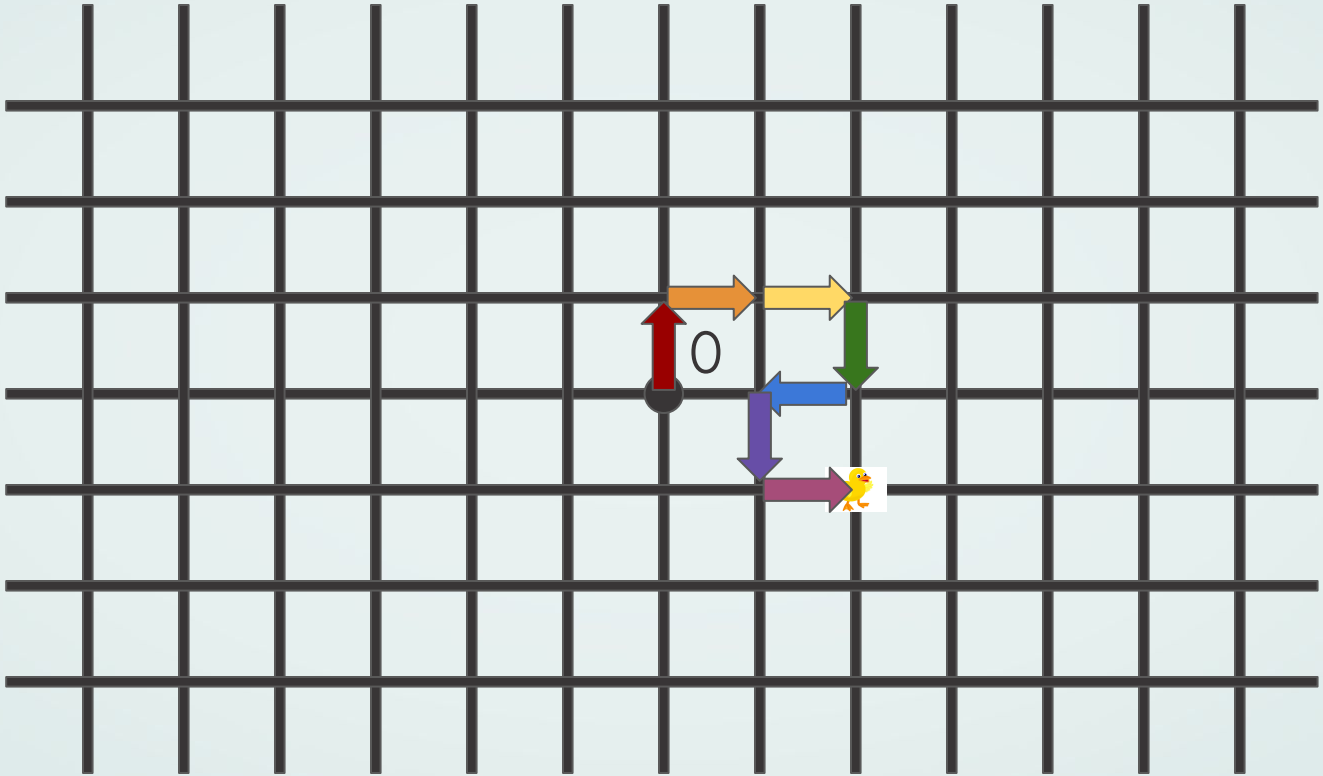

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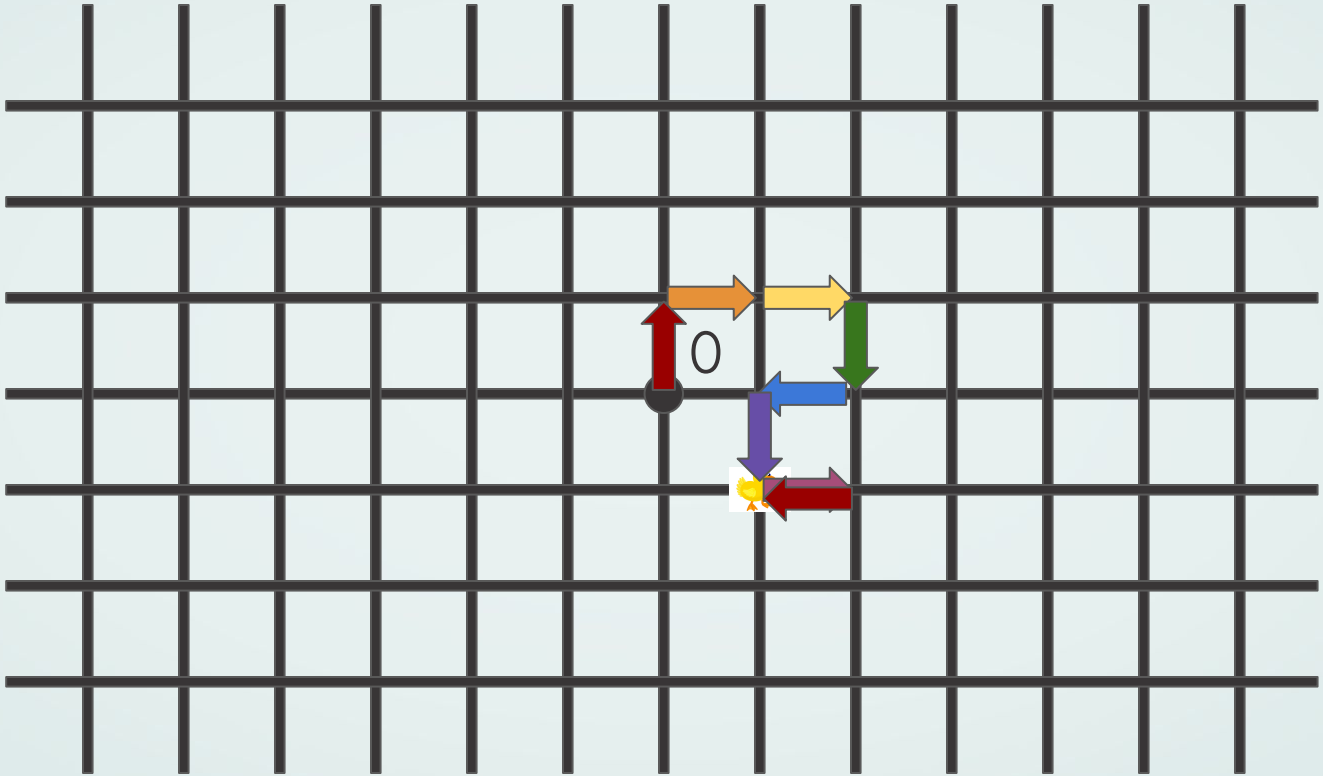

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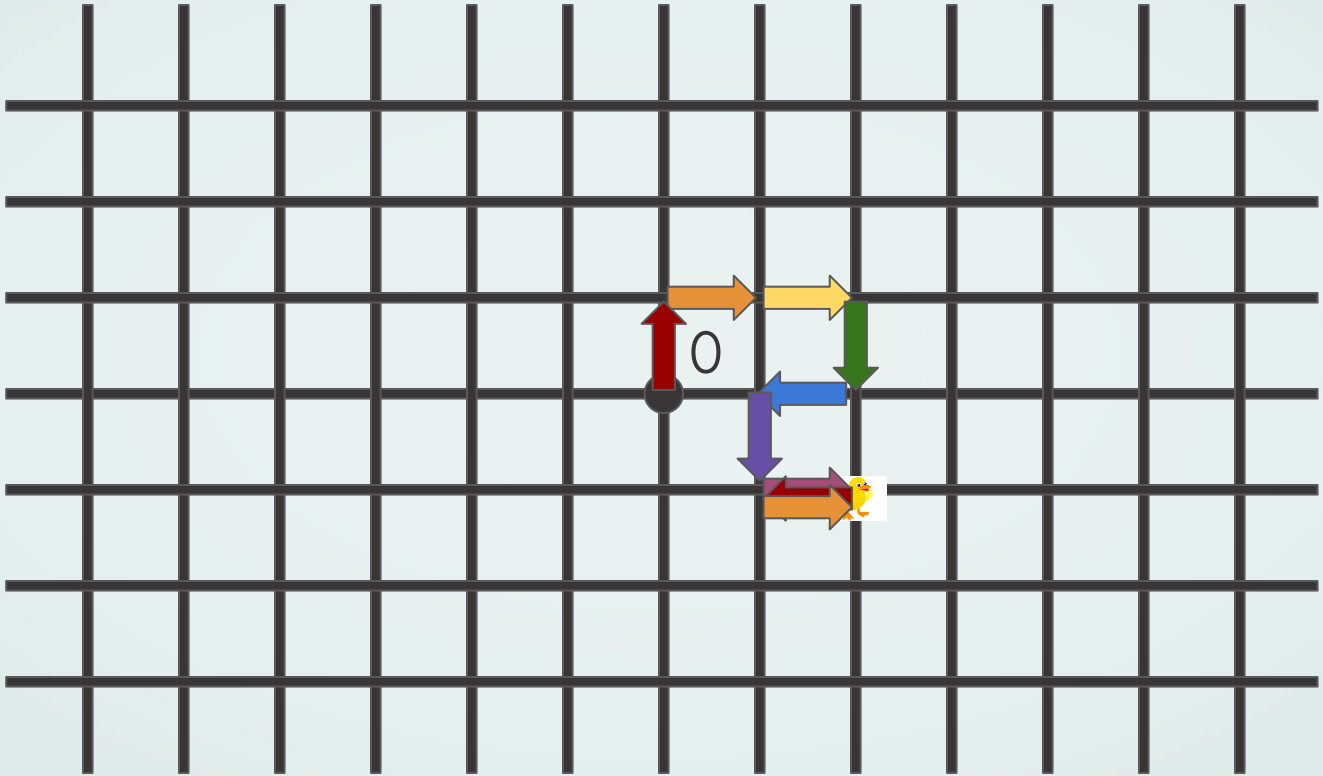

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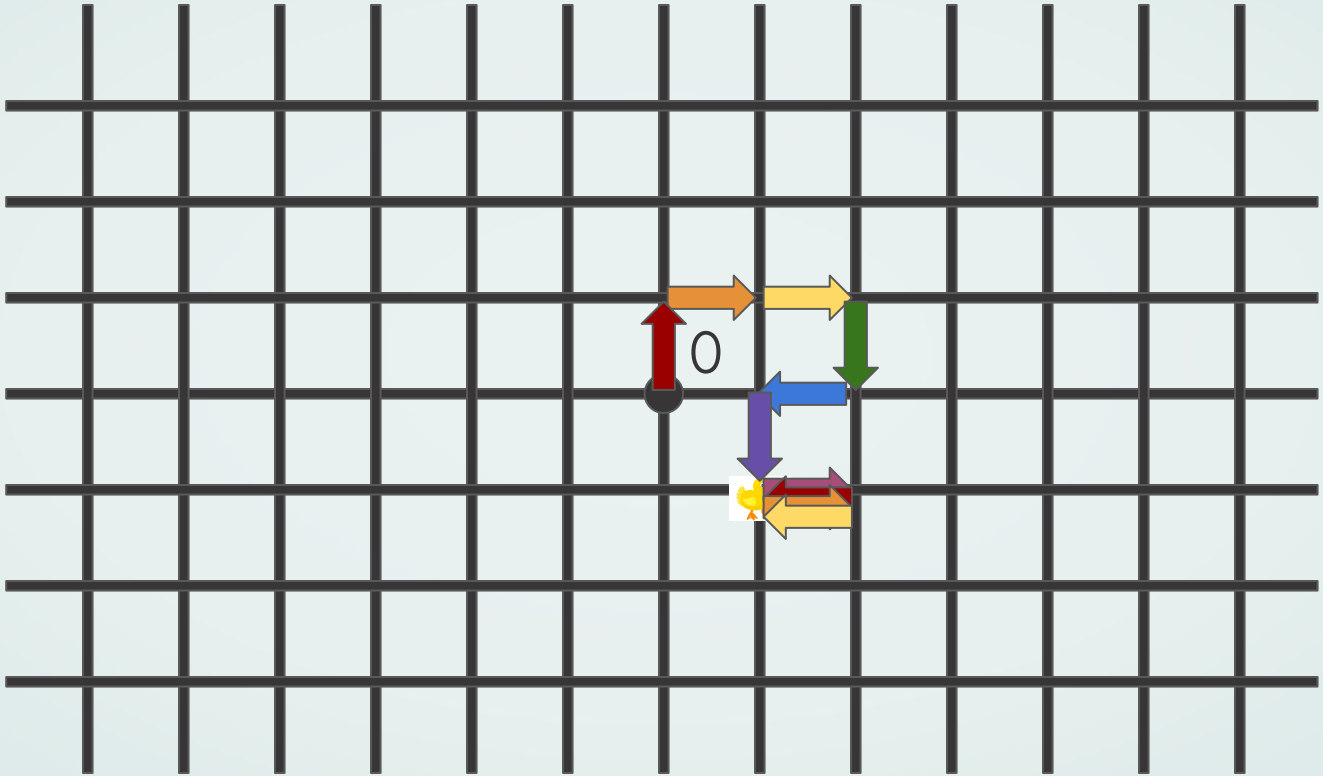

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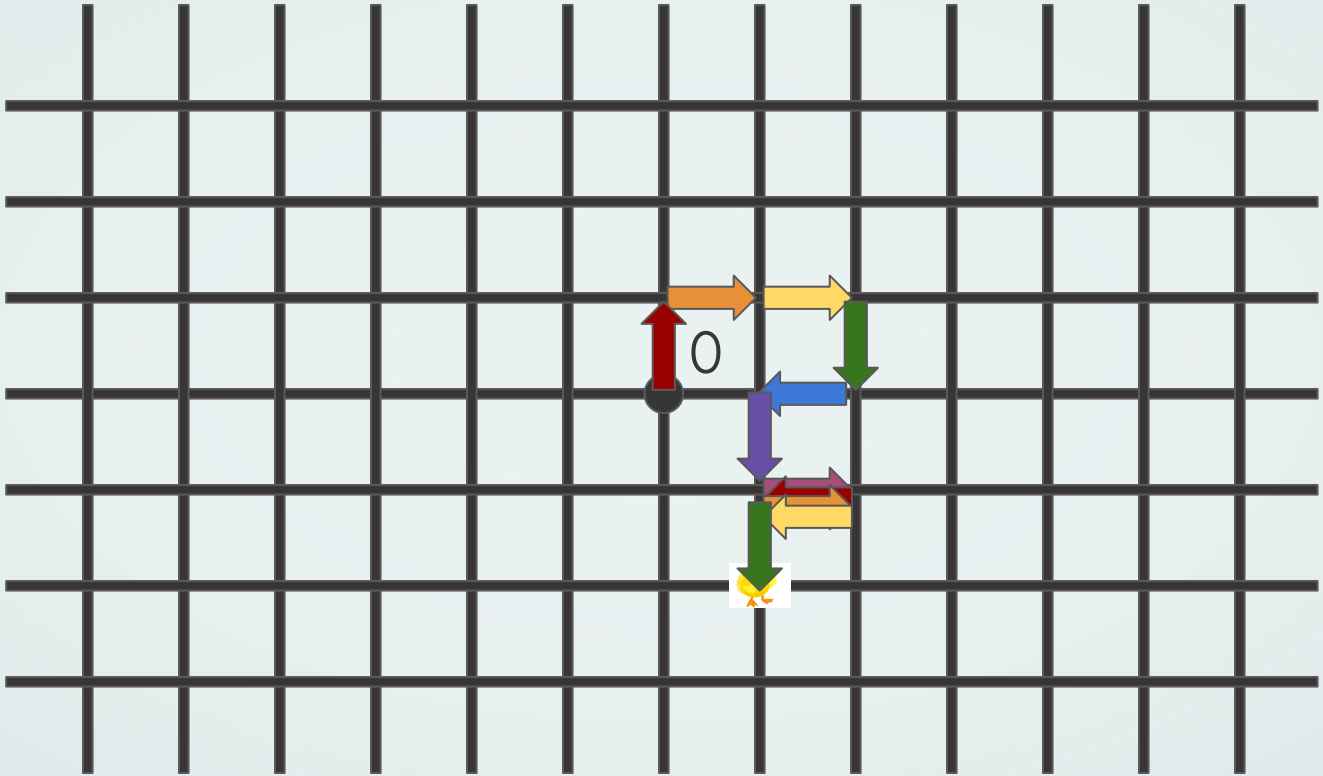

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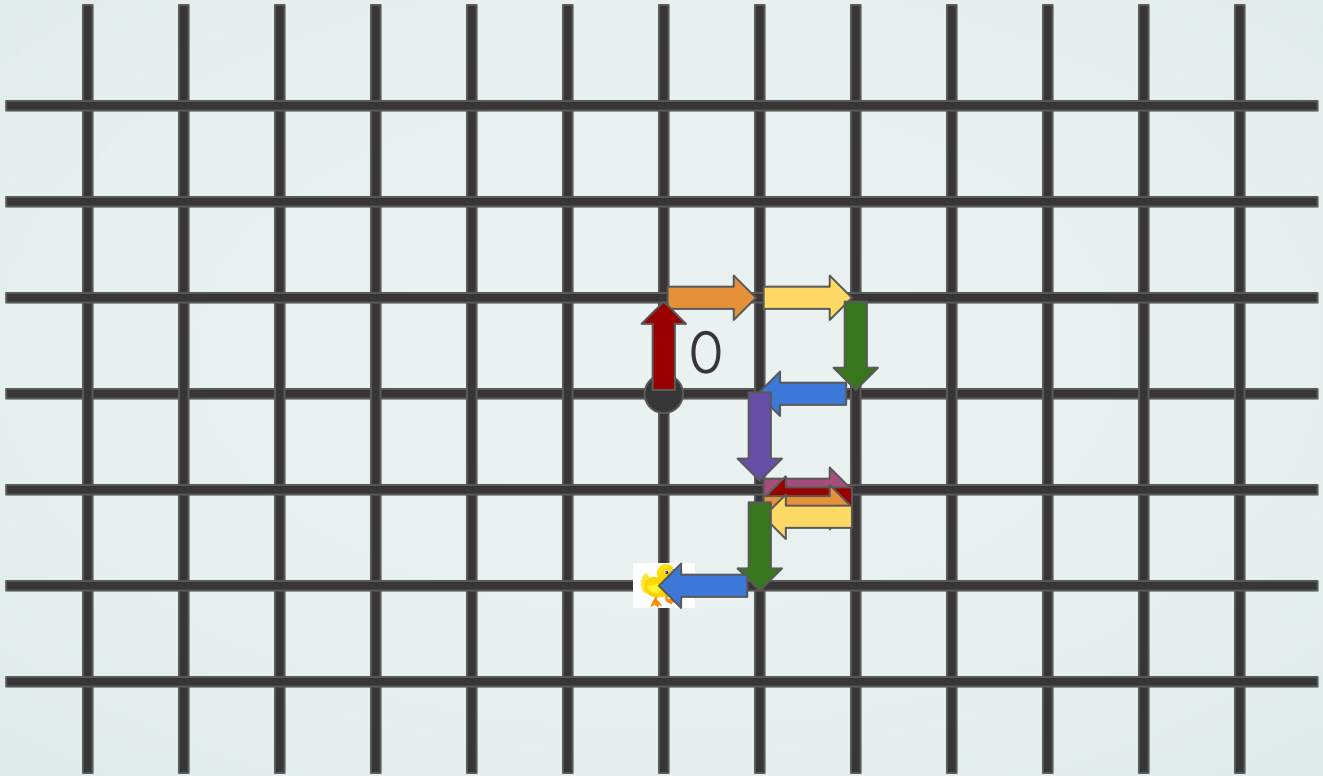

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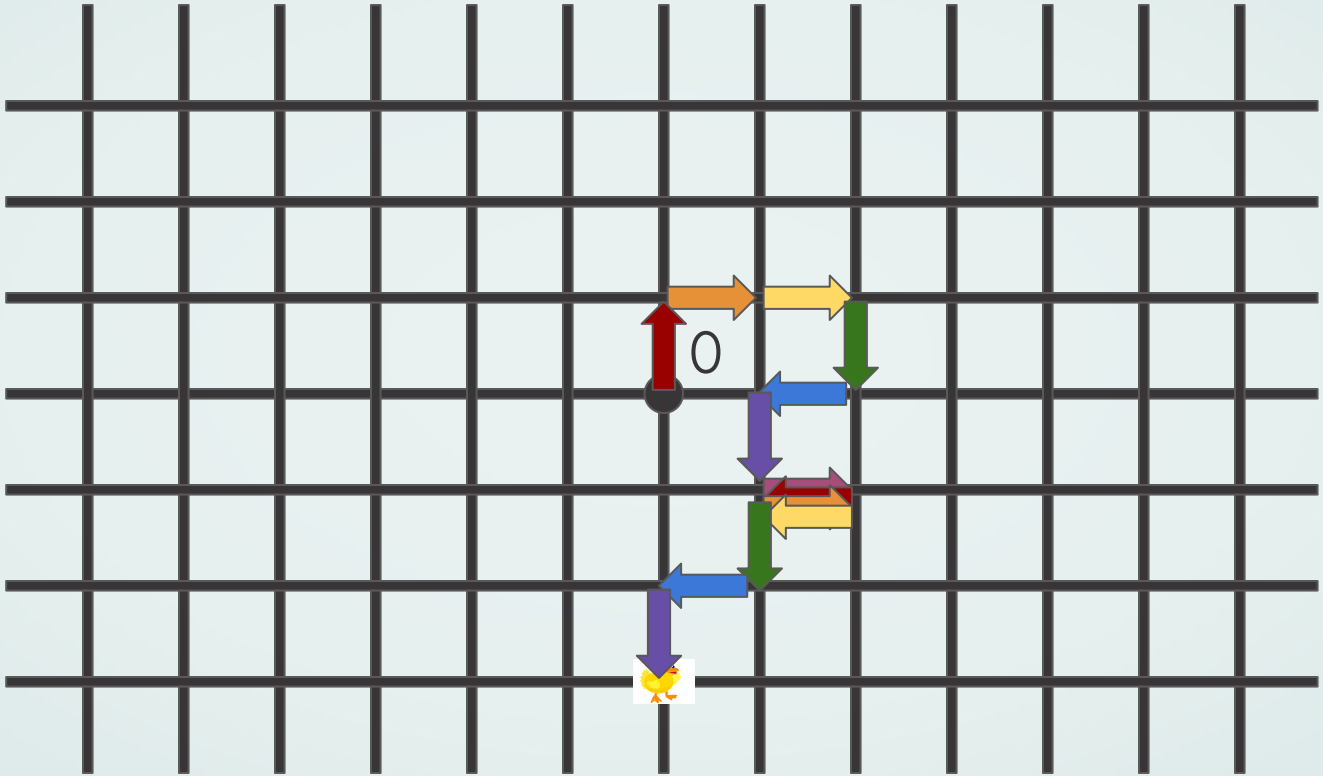

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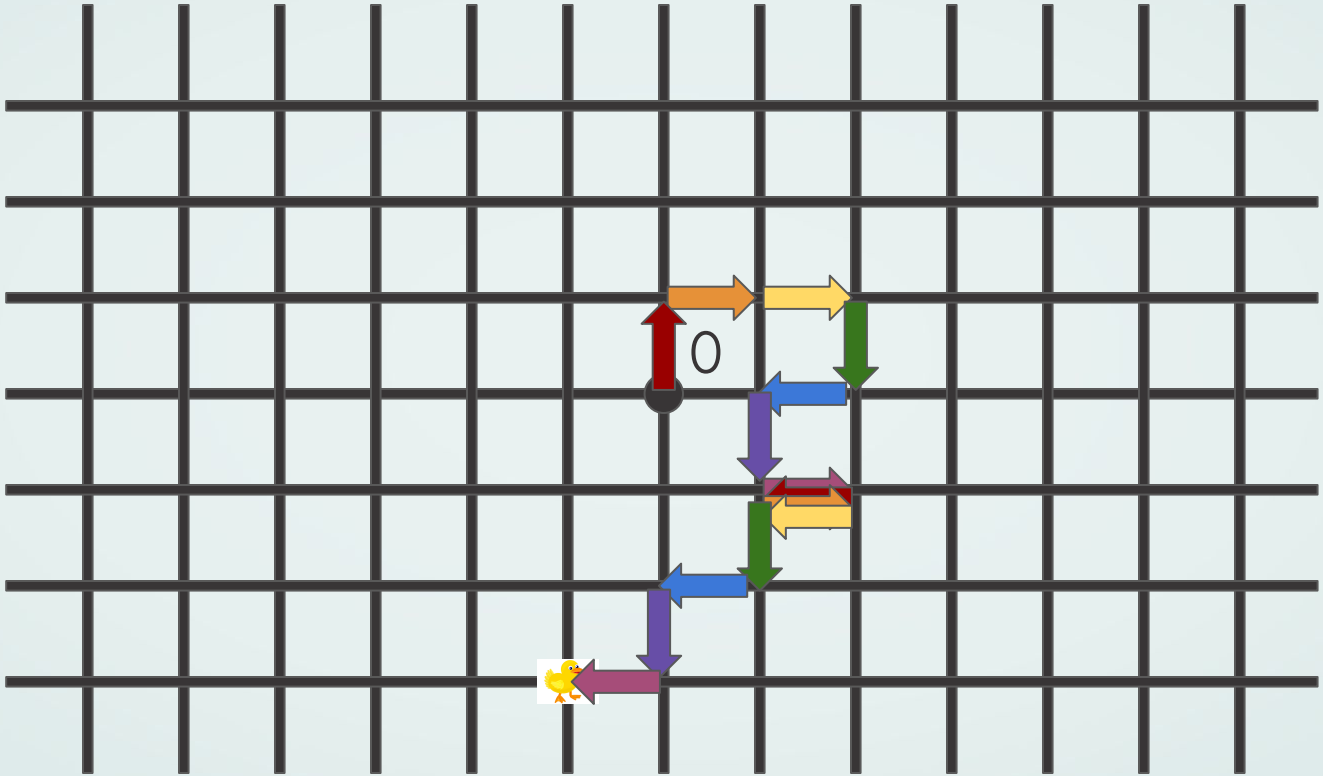
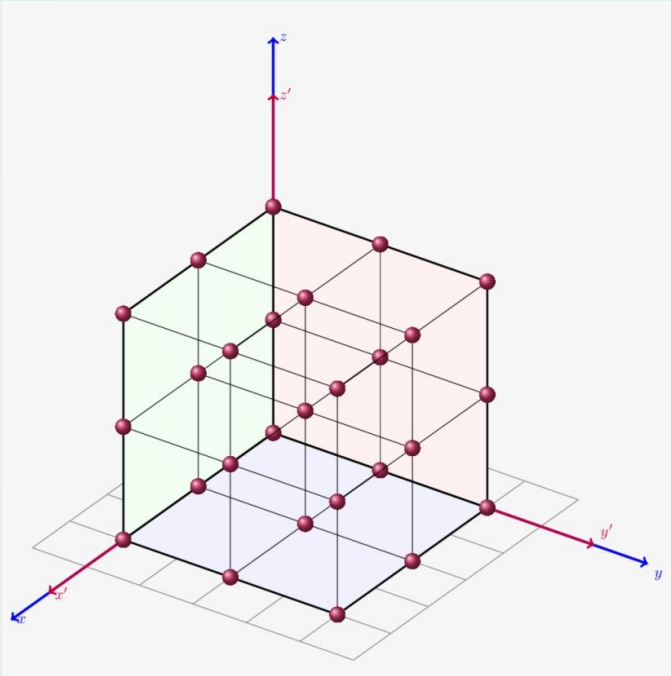


3D random walk: on a 3D grid



Returning home

A confused man will find his way home, but a confused bird may get lost forever

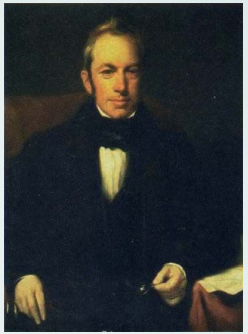
(paraphrased from Shizuo Kakutani)

Continuous Random Walks

And Brownian Motion

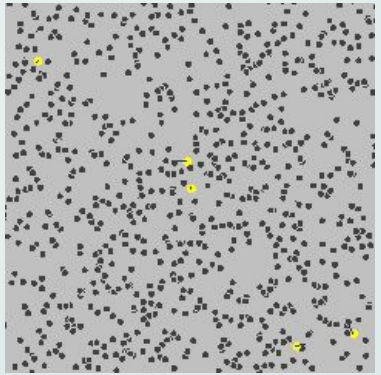
How do a pollen particles move in water?

- 1827: botanist Robert Brown observed movement of pollen grains in water w/ microscope



- Noted jittery movement of small pollen particles

- Did not know why this happened!

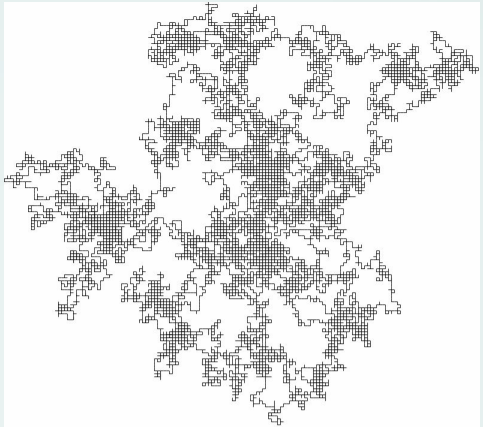


Further work towards A Mathematical Model

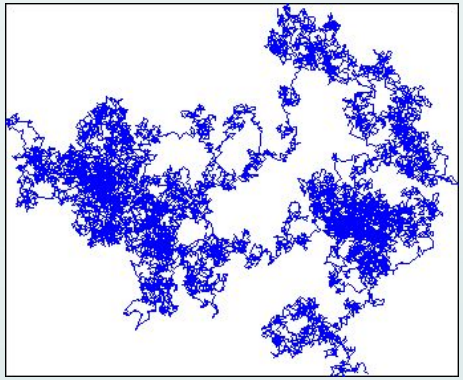
- Further study done by Louis Bachelier (finance) in 1900 and Albert Einstein (physics) in 1905
- Norbert Wiener (math) made rigorous mathematical model in 1923
- **Idea:** the particle's movement is a *continuous* random walk, with infinitely small steps

2D Continuous random walk / Brownian motion

A 2D random walk
with small steps



A 2D random walk
with very small steps



3D Continuous random walk / Brownian motion

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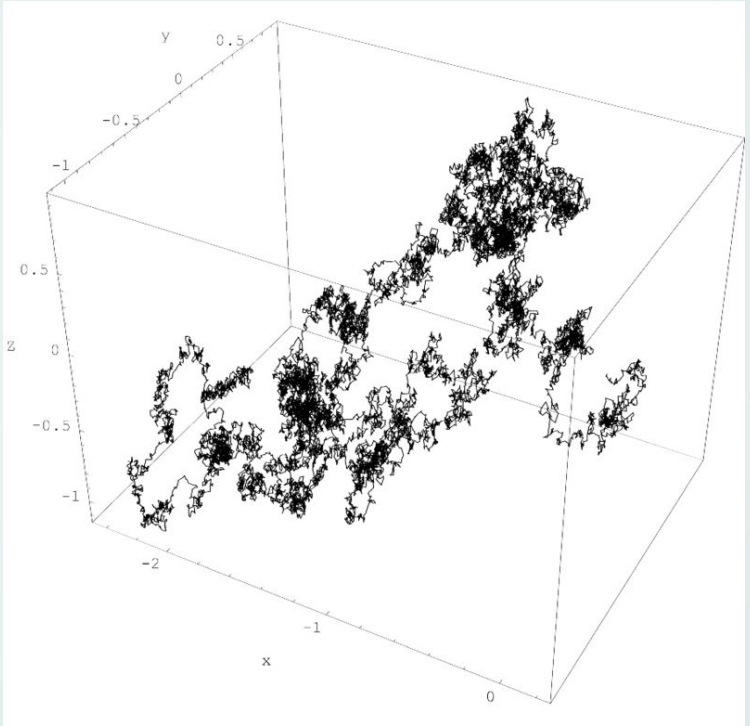


PageRank

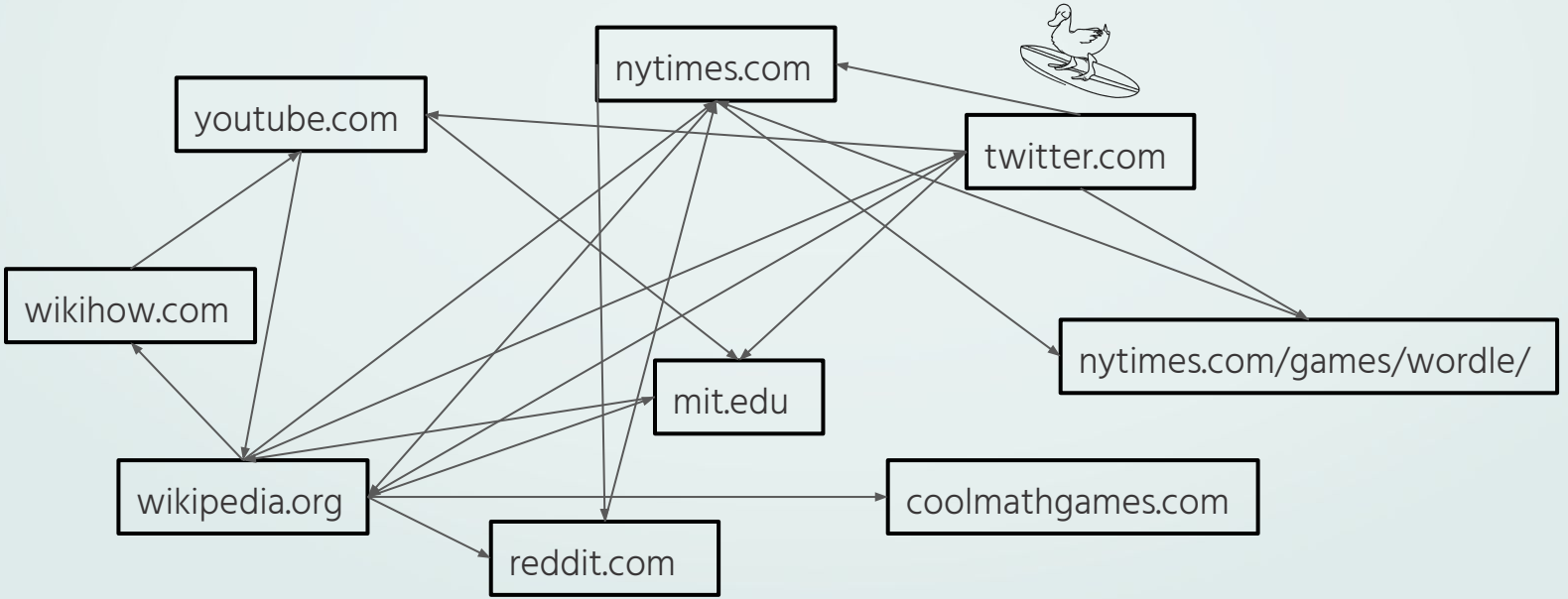
A Trillion Dollar Algorithm

Google's PageRank: The trillion dollar algorithm

- In the 90s, the World Wide Web grew to hundreds of millions of web pages
- Needed good search engine to find content on web
- 1996: Larry Page and Sergey Brin developed PageRank, founded Google in 1998 based on this algorithm

A Random Walk on the Web

Imagine a random surfer surfs the web by randomly clicking on links



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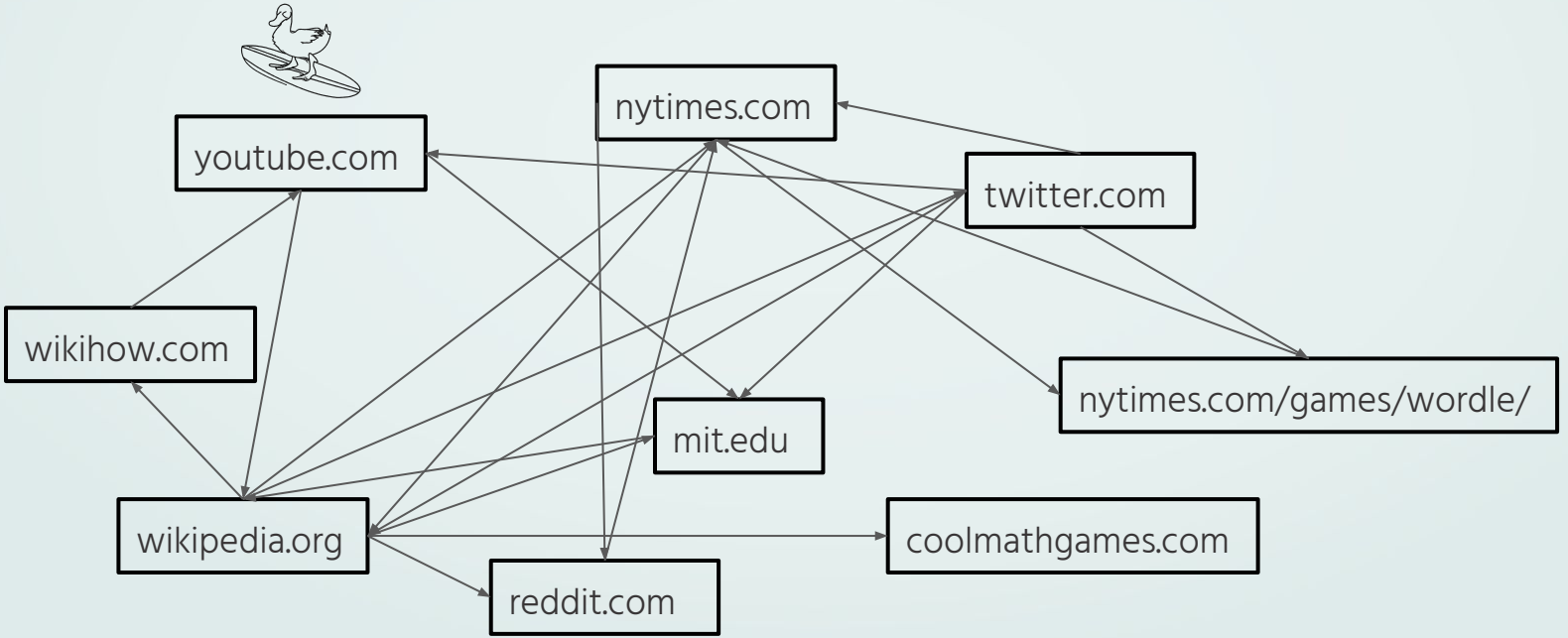

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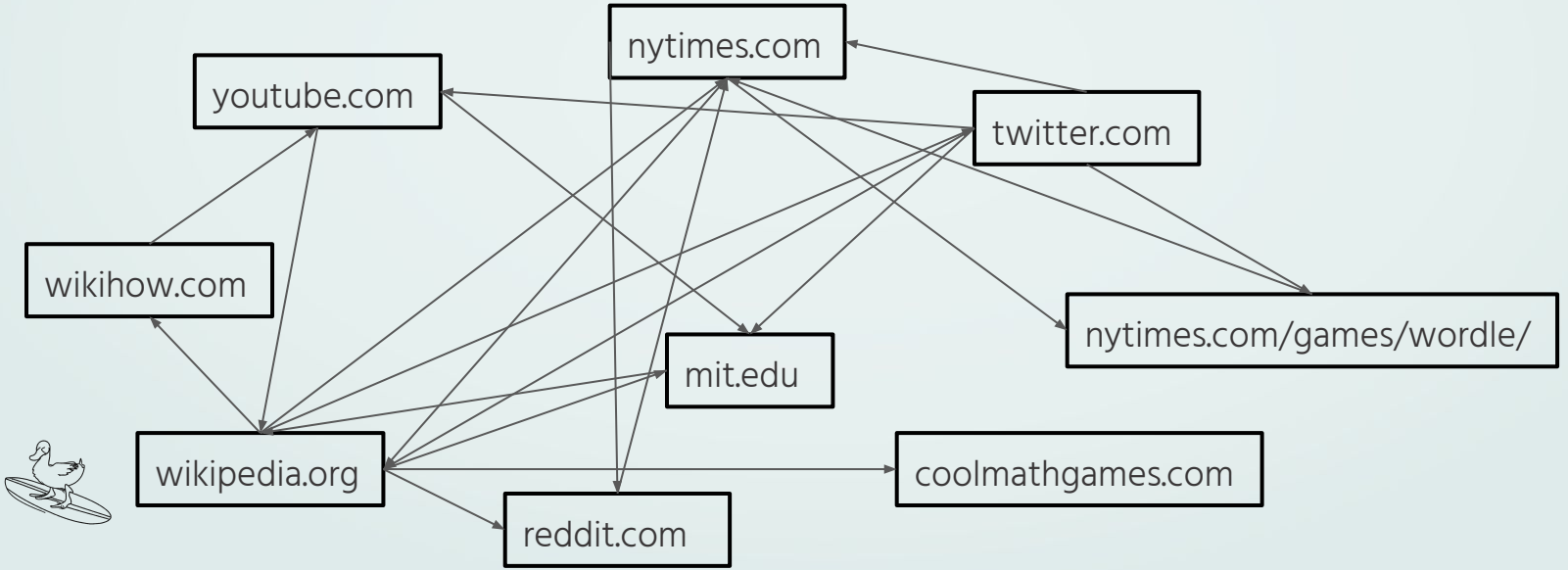

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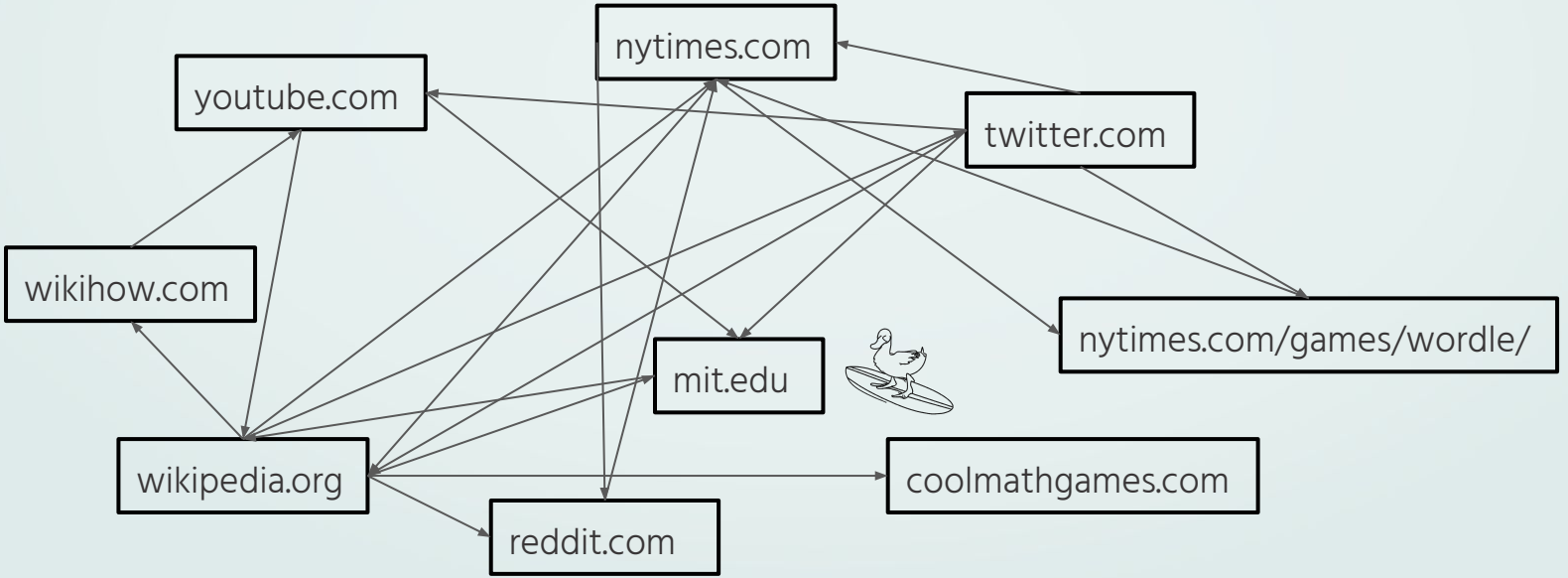


Which web pages are most important?

PageRank random walk interpretation: a page is highly ranked if the random surfer is likely to land on it

PageRank algorithm main idea:

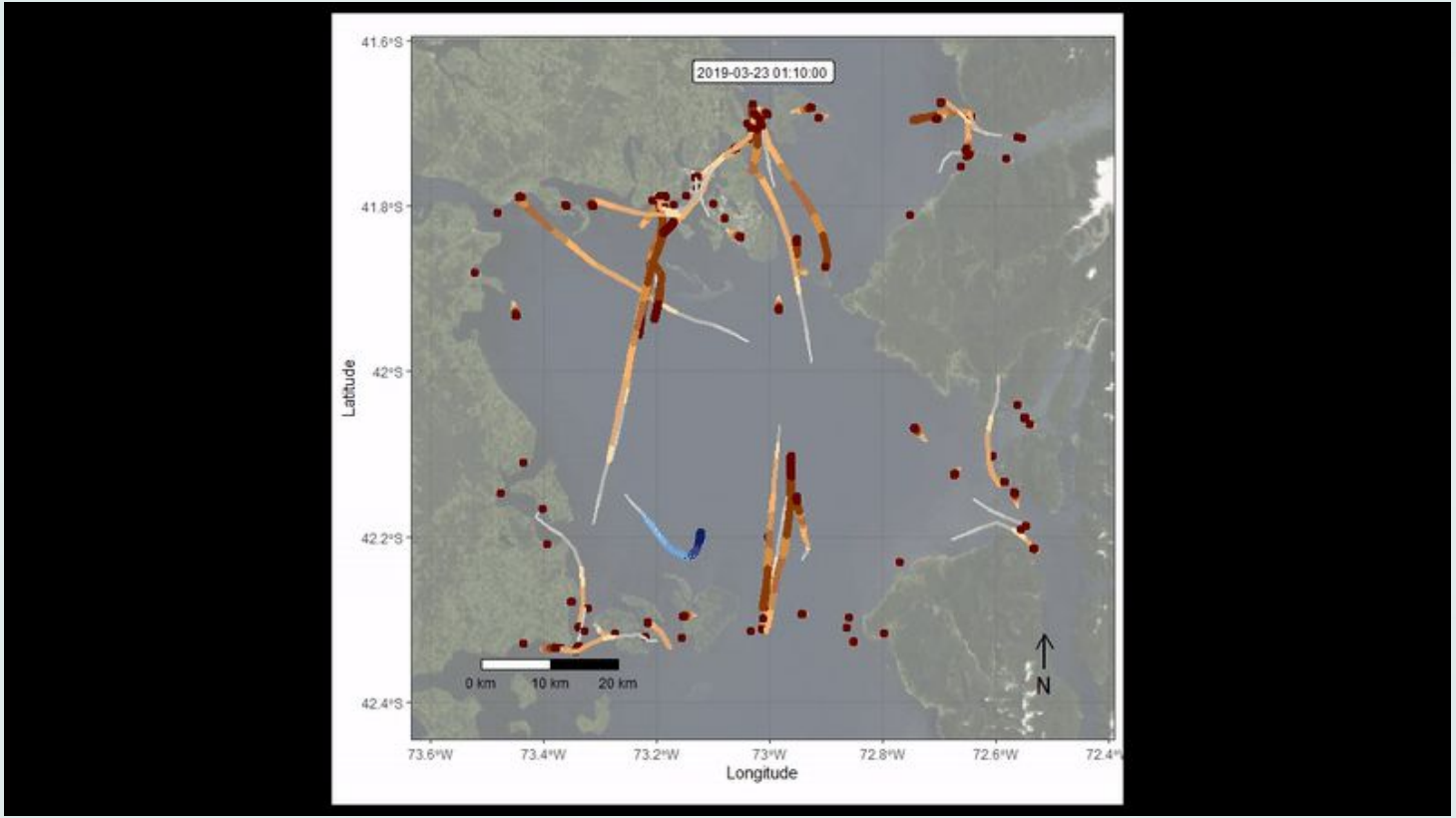
- Page A is important if an important page B links to it
- When is Page B important? By the same metric
- PageRank simultaneously solves for importances of all webpages (using linear algebra!)

Animal Motion

Modified Random Walks

Blue Whale Movement

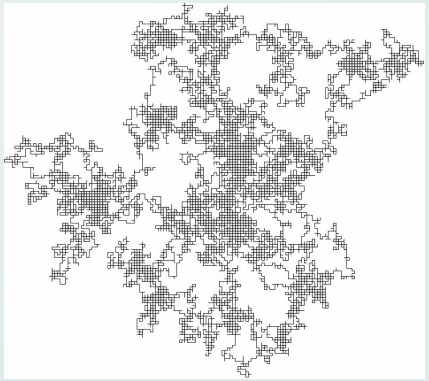
A Blue Whale Dodging Ships in Chile



Is a simple 2D / 3D random walk realistic?

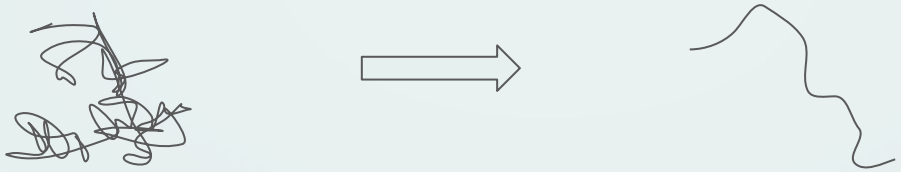
Not really:

- Animal movement is not that erratic
- Animals have goals when they move (e.g. find food, shelter, comfortable climate)
- There are obstacles in the real world



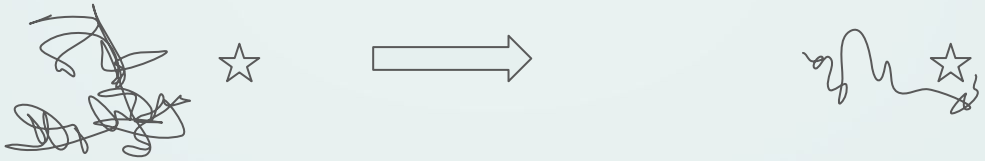
Random Walk Modification 1: Momentum

The random walker should have a tendency to move in the same direction it was already moving



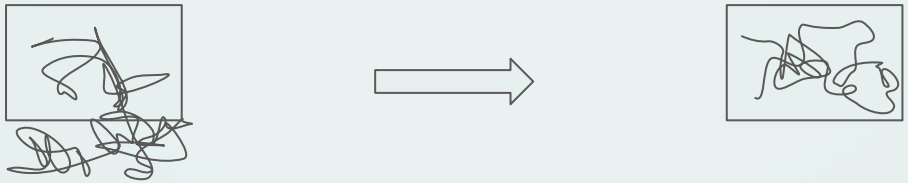
Random Walk Modification 2: Goals

The random walker should be more likely to move towards things that help them achieve their goals



Random Walk Modification 3: Terrain

The random walker should mostly stay within certain boundaries



Modelling Dragon Flight in Game of Thrones

Part of a project I did for the 2019 International Mathematical Competition in Modelling

Flying Over Westeros

- Momentum: dragons tend to fly in direction they are already going
- Goals: dragons like warmer temperatures more
- Terrain: dragons will not fly off the map of Westeros

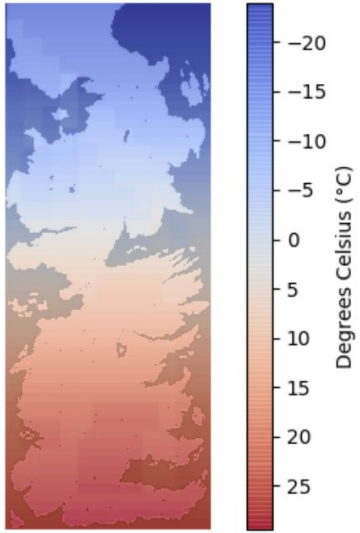
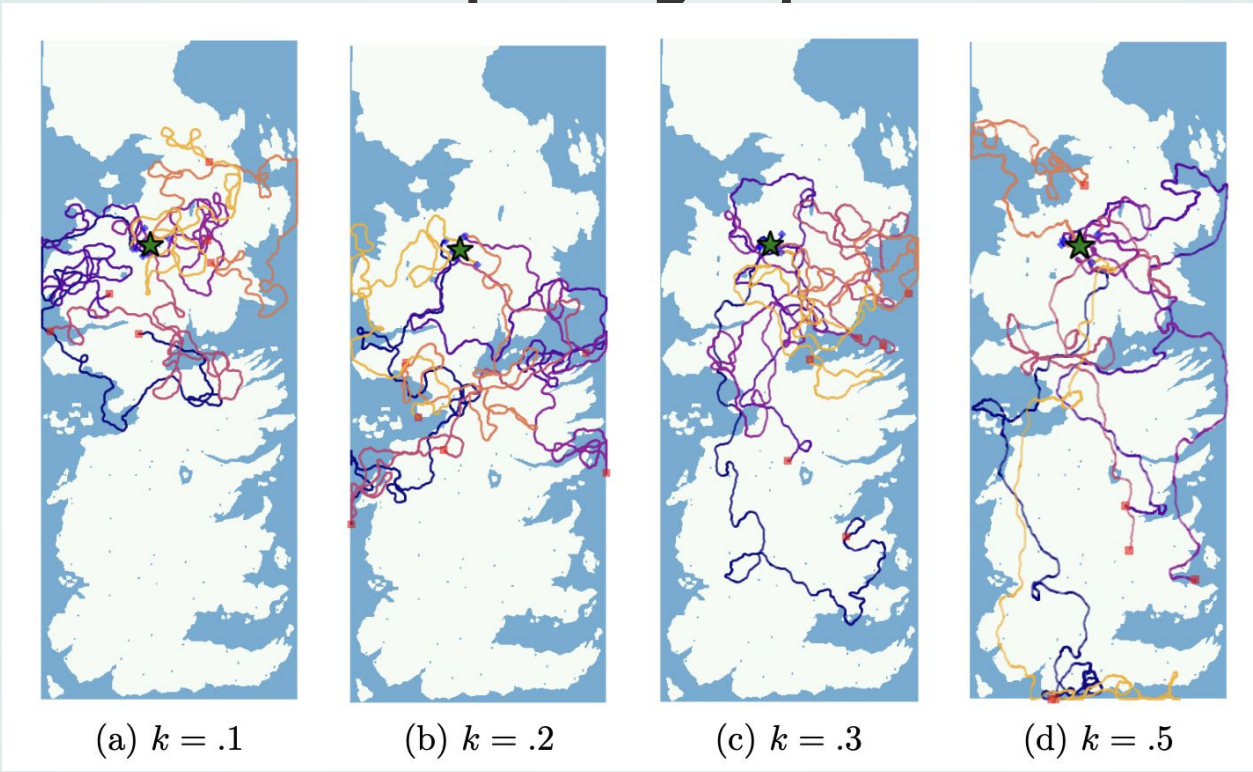


Figure 7: Temperature Map of Westeros during Winter. Temperatures approximately taken from (Tarly [18])

Sample flight paths



(a) $k = .1$ (b) $k = .2$ (c) $k = .3$ (d) $k = .5$

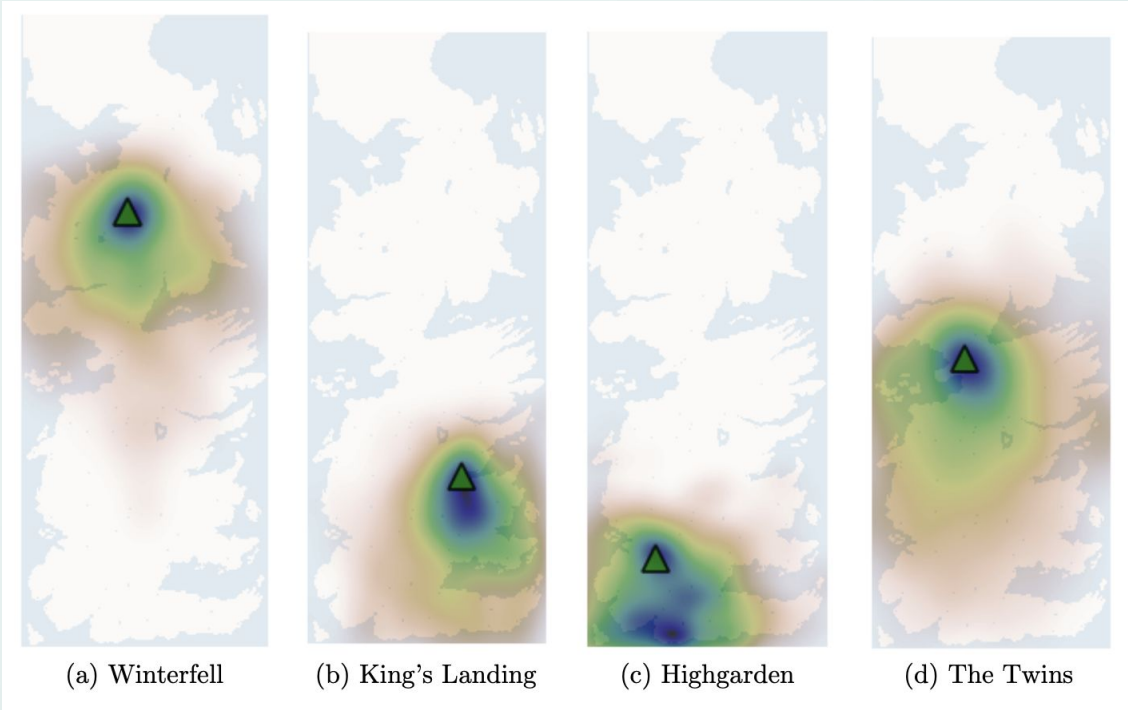
Higher k means preferring warmth more

Approximate home range

Triangle = home

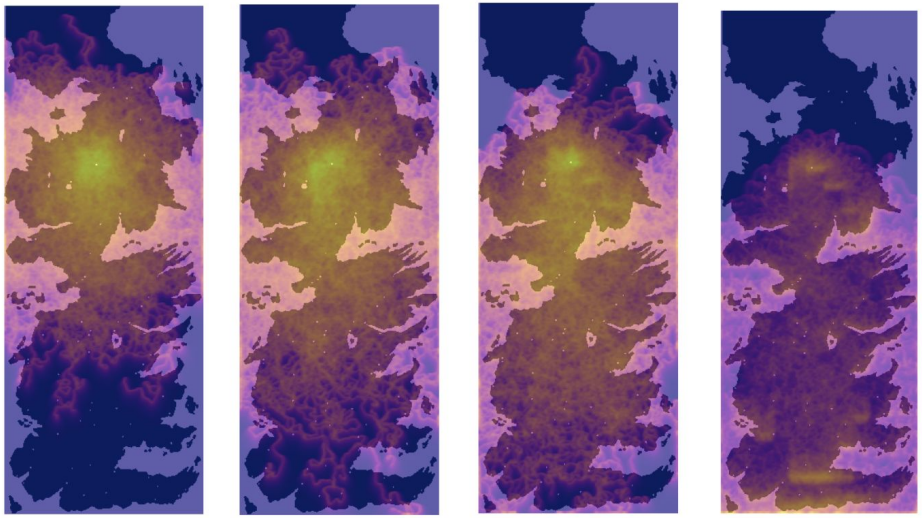
Shaded = places dragon will likely fly

Computed using statistical techniques



(a) Winterfell (b) King's Landing (c) Highgarden (d) The Twins

Heatmap of where dragons fly



(a) $k = .1$ (b) $k = .2$ (c) $k = .3$ (d) $k = .5$

Figure 11: Heatmaps of duration of flying over regions of the map. The more bright and yellow, the more time that that a dragon has spent in the area. 3500 random walks were simulated for each value of k.

Higher k means preferring warmth more

THANKS

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